On covariant long-distance modifications of Einstein theory and strong coupling problem

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Abstract

We present generic form of the covariant nonlocal action for infrared modifications of Einstein theory recently suggested within the weak-field curvature expansion. In the lowest order it is determined by two nonlocal operators — kernels of Ricci tensor and scalar quadratic forms. In models with a low strong-coupling scale this action also incorporates the strongly coupled mode which cannot be perturbatively integrated out in terms of the metric field. This mode enters the action as a Lagrange multiplier for the constraint on metric variables which reduces to the curvature scalar and enforces the latter to vanish on shell. Generic structure of the action is demonstrated on the examples of the Fierz-Pauli and Dvali-Gabadadze-Porrati models and their extensions. The gauge-dependence status of the strong-coupling and VDVZ problems is briefly discussed along with the manifestly gauge-invariant formalism of handling the braneworld gravitational models at classical and quantum levels.

1. Introduction: new mechanism of small cosmological constant

As is well known, the essence of the cosmological constant problem (CCP) consists in the enormous gap between the average density of energy in the modern Universe $\mathcal{E} \sim 10^{-29}\mathrm{g/cm}^3 \sim (10^{-5}\mathrm{eV})^4$, generating according to Einstein equations the cosmological acceleration with the Hubble constant H_0,

$$H_0^2 \sim G\,\mathcal{E}, \tag{1.1}$$

and the vacuum energy scale of all field theory models ranging from electroweak theory, $\mathcal{E} \sim (1\mathrm{TeV})^4$, to quantum gravity, $\mathcal{E} \sim (10^{19}\ \mathrm{GeV})^4$. Old attempts to resolve

this problem were based on building the models with zero vacuum energy, incorporating supersymmetry which protects the cosmological constant from renormalization. The cosmological constant mechanism in such models is based on cancellation of contributions of particles and their superpatners and stops working in the phase with spontaneously broken supersymmetry [1]. Moreover, it becomes completely meaningless in the framework of the cosmological acceleration phenomenon [2, 3, 4] for which the magnitude of the cosmological constant, though being very small, is still different from zero.

An alternative solution of CCP may be attributed to the curvature scalar sector of the Einstein-Hilbert action, rather than to its cosmological constant term. The smallness of H_0^2 in (1.1) can be explained not by small value of $\mathcal{E}$, but by the smallness of the proportionality coefficient — gravitational coupling constant G. In other words, the cosmological acceleration is so small not because the vacuum energy is small, but rather because the latter is gravitating very weakly. Special property of the vacuum energy as compared to other local sources of the gravitational field is the degree of its spacetime homogeneity. It is assumed that it is not clustering and practically homogeneous at the horizon scale,

$$\frac{\nabla\mathcal{E}}{\mathcal{E}} \sim H_0, \tag{1.2}$$

and gravitates with its long-wavelength gravitational constant $G_{LD} \ll G_P$ which is much smaller than the Planckian constant determining the everyday physics in the range of galaxies, planetary and solar systems, submillimeter Kavendish type experiments [5], etc., $H_0^2 \sim G_{LD}\mathcal{E} \ll G_P\mathcal{E}$.

This idea, that was apparently formulated in such explicit form for the first time in [6], represents the replacement of the fundamental gravitational constant in Einstein equations by the nonlocal operator, which for reasons of covariance can be regarded as a function of the d'Alembertian and which interpolates between the Planckian value of the gravitational constant and its long-distance magnitude[1]

$$G \Rightarrow G(\Box), \quad G_P > G(\Box) > G_{LD}. \tag{1.3}$$

Note that this mechanism of scale dependent gravitational constant is not unique. The notion of scale includes not only the degree of spacetime inhomogeneity, but also the field amplitude. Therefore, the infrared modification of the theory can also be based on the gravitational "constant" locally depending on some distinguished physical fields — sort of quintessence [1, 8]. However, such modifications are less universal because

[1]The idea of scale dependent Newton constant was also put forward in [7], though it was not formulated in terms of the nonlocal operator.

they are attributed to the behavior of concrete quintessence field, while the mechanism of the nonlocal replacement (1.3) leads to modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(\Box)\, T_{\mu\nu}, \tag{1.4}$$

in which independently of the field content of the matter source $T_{\mu\nu} = T_{\mu\nu}(x)$ the gravitational strength of the latter is determined solely by the degree of its inhomogeneity in x.

One of the first problems associated with (1.4) is the lack of general covariance — no covariant action can generate such equations of motion with a nontrivial $G(\Box)$. The solution of this problem was suggested in [9] by viewing (1.4) only as a first, linear in the curvature, approximation for correct equations of motion, their covariant action being constructed as a weak field expansion in powers of the curvature with nonlocal coefficients. The most solid example of the theory incorporating the mechanism (1.3) is the Dvali-Gabadaze-Porrati (DGP) model of brane-induced gravity [10] which is very interesting due to the fact that it naturally contains the mechanism of the cosmological acceleration [11]. However, this model turned out to be suffering from the strong-coupling problem [12, 13, 14] which invalidates the curvature expansion theory, makes it inefficient without the fundamental UV completion at the quantum level and features the van Dam-Veltman-Zakharov (VDVZ) discontinuity problem [15]. Though this problem does not indicate the physical inconsistency of the underlying theory and is likely to be circumvented in tree-level applications by accounting for nonlinear effects [16] or using the relativistic expansion [17], in quantum domain it rises at full height, because of the essentially perturbative nature of Feynman loop expansion. Therefore, its status becomes very important for possible generalizations of the Einstein theory.

The goal of this paper is to discuss the most general version of the covariant nonlocal action for long-distance modifications of Einstein theory, suggested in [9]. It turns out that in the lowest (quadratic in curvature) order it is determined by two nonlocal operators, kernels of Ricci and scalar curvature quadratic forms, and also may contain additional term responsible for the strong-coupling problem. The strongly coupled mode enters this term linearly like the Lagrangian multiplier of the auxiliary constraint on metric field and cannot be perturbatively integrated out in terms of metric variables. After discussing this in Sect.2 we demonstrate generic structure of the action on the examples of Fierz-Pauli and DGP models, consider the details of VDVZ and strong-coupling problems and review their generalizations which allow one to circumvent these problems. In particular, in Sect.5 we review a recently suggested constrained perturbation theory in the DGP model [18] and show that it turns out to be a noncovariant modification of the original DGP model. In the concluding section we briefly discuss the gauge independent status of the VDVZ and strong-coupling problems and

outline the manifestly gauge-invariant technique for classical and quantum braneworld models.

2. Covariant nonlocal action for infrared modifications of Einstein theory

The idea of replacing the gravitational constant by a function of the covariant d'Alembertian $\Box = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$, according to [6] consists in the following modification of the left hand side of Einstein equations

$$\frac{M^2(\Box)}{16\pi}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = \frac{1}{2}\,T_{\mu\nu}, \tag{2.1}$$

where the nonlocal Planck mass is a function of the dimensionless combination of $\Box$ and additional length scale L, interpolating between the Planck constant for matter sources of small size $\ll L$ and long distance constant $G_{LD} = G(0)$,

$$\frac{1}{G(\Box)} \equiv M^2(\Box) = M_P^2\,(1 + \mathcal{F}(L^2\Box\,)). \tag{2.2}$$

If the function of $z = L^2\Box$ satisfies the conditions $\mathcal{F}(z) \to 0$ at $z \gg 1$ and $\mathcal{F}(z) \to \mathcal{F}(0) \gg 1$ at $z \to 0$, then the infrared modification is inessential for processes varying in spacetime faster than $1/L$ and vicy versa — becomes significant — for slow phenomena with wavelengths $\sim L$ and higher.

An obvious difficulty with the construction of the above type is that for any nontrivial operator $\mathcal{F}(L^2\Box\,)$ the left hand side of (2.1) does not satisfy the Bianchi identities and cannot be obtained by varying the covariant action. In particular, a naive attempt to modify the gravitational action according to

$$M_P^2\int dx\,g^{1/2}R \Rightarrow \int dx\,g^{1/2}M^2(\Box)R = M^2(0)\int dx\,g^{1/2}R \tag{2.3}$$

is meaningless, because after the integration by parts the covariant d'Alembertian when acting to the left (on 1) picks up its zero mode and the nonlocal operator in all regimes reduces to its infrared value $M_P^2(0)$.

This problem can be solved within the assumption of weak field approximation implying that the equation (2.1) represents only the first linear term of the perturbation expansion in powers of the curvature. Its left hand side must include higher orders in the curvature, the nonlocal gravitational action $S_{NL}[\,g_{\mu\nu}\,]$ generating the modified equations according to

$$\frac{\delta S_{NL}[\,g\,]}{\delta g_{\mu\nu}(x)} = \frac{M^2(\Box)}{16\pi}\,g^{1/2}\,\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right) + \mathrm{O}\,[\,R_{\mu\nu}^2\,]. \tag{2.4}$$

To obtain the leading term of $S_{NL}[\, g_{\mu\nu}\,]$, this equation can be functionally integrated in the explicit form [9] with the aid of the covariant curvature expansion technique of [19]. The essence of this technique consists in the possibility to convert noncovariant series in powers of gravitational perturbations $h_{\mu\nu}$ into the series of spacetime curvature and its derivatives with the covariant nonlocal coefficients. The starting point is the expansion of the Ricci tensor

$$R_{\mu\nu} = -\frac{1}{2}\,\Box\, h_{\mu\nu} + \frac{1}{2}\Big(\nabla_\mu F_\nu + \nabla_\nu F_\mu\Big) + \mathrm{O}\,[\,h^2_{\mu\nu}\,], \tag{2.5}$$

($F_\mu \equiv \nabla^\lambda h_{\mu\lambda} - \frac{1}{2}\nabla_\mu h$ is the linearized de Donder-Fock or harmonic gauge), which can be solved by iterations with respect to $h_{\mu\nu}$ in the form of the nonlocal expansion in the curvature, beginning with

$$h_{\mu\nu} = -\frac{2}{\Box} R_{\mu\nu} + \nabla_\mu f_\nu + \nabla_\nu f_\mu + \mathrm{O}\,[\,R^2_{\mu\nu}\,]. \tag{2.6}$$

Here $\nabla_\mu f_\nu + \nabla_\nu f_\mu$ reflects the gauge arbitrariness in this solution originating from the terms with a harmonic gauge in (2.5).

As a result, the nonlocal action generating (2.4) begins with the quadratic order in the curvature [9]

$$S_{NL}[\, g_{\mu\nu}] = -\frac{1}{16\pi}\int dx\, g^{1/2}\left\{\Big(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\Big)\frac{M^2(\Box)}{\Box}\,R_{\mu\nu} + \mathrm{O}[R^3_{\mu\nu}]\right\}. \tag{2.7}$$

Interestingly, in the simplest case of constant $M^2(\Box) = M_P^2$ it should reproduce the Einstein-Hilbert action, which looks puzzling because it does not at all contain the term linear in the curvature. The explanation of this paradox consists in the observation that the Einstein action in the asymptotically-flat spacetime with the metric behaving as $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$, $h_{\mu\nu} = \mathrm{O}\,(1/|x|^2)$, $|x| \to \infty$, includes the Gibbons-Hawking surface integral over spacetime infinity

$$S_E[\, g_{\mu\nu}] = -\frac{M_P^2}{16\pi}\int dx\, g^{1/2}\, R(\,g\,) + \frac{M_P^2}{16\pi}\int_{|x|\to\infty} d\sigma^\mu\, \Big(\partial^\nu h_{\mu\nu} - \partial_\mu h\Big). \tag{2.8}$$

This integral can be transformed to the form of the volume integral of $\partial^\mu(\partial^\nu h_{\mu\nu} - \partial_\mu h)$ — linear in $h_{\mu\nu}$ part of the scalar curvature — and, with the aid of (2.6), covariantly expanded in powers of the curvature. Up to quadratic terms inclusive this expansion has the form [9]

$$\int_{|x|\to\infty} d\sigma^\mu\Big(\partial^\nu h_{\mu\nu} - \partial_\mu h\Big) = \int dx\, g^{1/2}\left\{R - \Big(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\Big)\frac{1}{\Box}R_{\mu\nu} + \ldots\right\}. \tag{2.9}$$

Therefore, on substituting to (2.8) linear in Ricci scalar terms cancel out, and the quadratic terms reproduce the expression (2.7) with the coefficient $M^2(\Box) = M_P^2$ which

can be pulled out of the integral. The result is the *nonlocal* form of the *local* Einstein action [20, 21, 9],

$$S_E[\,g_{\mu\nu}] = -\frac{1}{16\pi}\int dx\, g^{1/2}\left\{\Big(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\Big)\frac{M_P^2}{\Box}\,R_{\mu\nu} + \mathrm{O}[R_{\mu\nu}^3]\right\}. \tag{2.10}$$

The fact that this action begins with the quadratic order in $R_{\mu\nu} \sim h_{\mu\nu}$ obviously corresponds to the theory of massless spin-2 field. Less trivial is the nonlocality of this action, which is the price one should pay for the manifest covariance of this expansion in contrast to the local in terms of $h_{\mu\nu}$, but noncovariant action for symmetric spin-2 tensor field.

Thus the action (2.7) serves as a direct realization of the idea of nonlocal gravitational coupling constant — replacement of M_P^2 in the nonlocal version of the Einstein action (2.10) by the operator (2.2) as suggested in [9]. However, there is a question (that was not addressed in [9]) to what an extent this infrared modification is generic even in the quadratic order approximation? It is obvious that in the general case the operator kernels in the quadratic Ricci tensor and scalar forms can be different, so that the generalization of (2.7) takes the form

$$S_{NL}[\,g_{\mu\nu}] = -\frac{1}{16\pi}\int dx\, g^{1/2}\left\{R^{\mu\nu}\frac{M_1^2(\Box)}{\Box}R_{\mu\nu} - \frac{1}{2}\,R\frac{M_2^2(\Box)}{\Box}R + \mathrm{O}\,[\,R_{\mu\nu}^3\,]\right\}, \tag{2.11}$$

where the two nonlocal "masses" should tend to one and the same Planckian limit M_P only in the short and intermediate distance domain,

$$M_{1,2}^2(\Box) \to M_P^2, \quad \Box \gg 1/L. \tag{2.12}$$

The last requirement implies the recovery of the Einstein general relativity at intermediate distances including, in particular, the absence of the VDVZ discontinuity problem — correct tensor structure of the gravitational potential. For the theory (2.11) the linear potential generated by the conserved matter sources $T_{\mu\nu}$ has the form

$$h_{\mu\nu} = -\frac{16\pi}{M_1^2(\Box)\Box}\,\Big(T_{\mu\nu} - \frac{1}{2}\,\alpha(\Box)\,\eta_{\mu\nu}\,T\Big) + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu. \tag{2.13}$$

Here the longitudinal terms correspond to gauge arbitrariness and the operator coefficient $\alpha(\Box)$ equals

$$\alpha(\Box) = \frac{2M_2^2(\Box) - M_1^2(\Box)}{3M_2^2(\Box) - 2M_1^2(\Box)}. \tag{2.14}$$

Generically, it should take the general relativistic value $\alpha = 1$ only in the limit $M_1^2(\Box) \to M_2^2(\Box)$ accounting for the restriction (2.12).

Expression (2.11) seems to give the most generic form of the gravitational theory in weak field approximation, dictated by general covariance. However, this is also not the end of the story — infrared limit of the theory together with spin-2 polarization can accommodate additional degrees of freedom that are not taken into account in this expression. One would think that, under a natural assumption that these degrees of freedom are not directly coupled to matter, they can be integrated out, which reduces to additional contributions to $M_1^2(\Box)$ and $M_2^2(\Box)$. However, such a reduction is always possible except the case when the equations of motion for additional fields cannot be solved for them in terms of the metric. This happens when these fields enter the action linearly and play the role of Lagrange multipliers of certain combinations of metric variables. Unfortunately, such fields persist at higher orders of perturbation theory and, thus, give rise to the problem of the low strong-coupling scale. This precludes from using the conventional perturbation theory in a wide range of distances very well probed by numerous Kavendish type and solar system experiments. Below we demonstrate this phenomenon on the examples of the Fierz-Pauli (FP) theory and the effective 4-dimensional theory of brane induced gravity of Dvali-Gabadadze-Porrati (DGP).

3. Fierz-Pauli model and VDVZ problem

The simplest infrared modification of the Einstein theory is represented by the Fierz-Pauli model of massive tensor field. It is described by the quadratic part of the Einstein action (2.8) modified by the *noncovariant* mass term on the *flat*-spacetime background

$$S_{\rm mass}[\,g_{\mu\nu}] = -\frac{M_P^2}{16\pi}\int d^4x\,\left(\frac{m^2}{4}\,h_{\mu\nu}^2 - \frac{m^2}{4}\,h^2\right), \tag{3.1}$$

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}, \quad h \equiv \eta^{\mu\nu} h_{\mu\nu}. \tag{3.2}$$

This is the only Lorentz-covariant combination of mass terms which guarantees the absence of ghosts in the theory. Linear equations of motion in this model have the form

$$R_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}\,R + \frac{m^2}{2}\,(h_{\mu\nu} - \eta_{\mu\nu}h) = 8\pi G_4\,T_{\mu\nu}, \tag{3.3}$$

where under $R_{\mu\nu}$ we understand the part of Ricci tensor linear in the graviton field. Differentiating this equation and taking into account the linearized Bianchi identity and the concservation of $T_{\mu\nu}$ we obtain the "gauge" for $h_{\mu\nu}$

$$\partial^\mu(h_{\mu\nu} - \eta_{\mu\nu}h) = 0, \tag{3.4}$$

which as a corollary has the vanishing of the linearized curvature scalar

$$R = \partial^\mu \partial^\nu h_{\mu\nu} - \Box h = 0 \tag{3.5}$$

(note that this equation is satisfied even for nonvanishing trace of matter stress tensor). As a result the gravitational field generated by the matter source takes the form

$$h_{\mu\nu} = -16\pi G_4 \, \frac{1}{\Box - m^2} \, \Big(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T \Big) + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \tag{3.6}$$

up to the longitudinal terms which do not couple to the conserved matter sources. Since the FP theory is not gauge invariant, these terms are not arbitrary and determined by the fixed vector

$$\xi_\mu = -\frac{8\pi G_4}{3m^2} \frac{1}{\Box - m^2} \partial_\mu T, \tag{3.7}$$

which, in particular, guarantees the validity of Eq. (3.4).

Tensor structure of massive graviton field here differs from the general relativistic case — the trace of $T_{\mu\nu}$ in Einstein theory has a coefficient 1/2 rather than 1/3 in FP model. This discrepancy remains also in the limit of vanishing mass and constitutes the VDVZ discontinuity problem [15]: massless limit of FP model does not reproduce the predictions for massless graviton of Einstein theory. This problem originates from the additional degree of freedom which is missing in Einstein theory and, on the contrary, exists in FP model for all values of the graviton mass. From the viewpoint of the general framework for infrared modifications of Einstein theory, this degree of freedom should enter the action (2.11) as a Lagrange multiplyer responsible for the auxiliary equation (3.5). Without it the equations of motion following from (2.11) do not reproduce the tensor structure (3.6) with any choice of operators $M_1^2(\Box)$ and $M_2^2(\Box)$. Indeed, the coefficient (2.14) takes a numerical value of FP model, $\alpha_{FP} = 1/3$, only in a singular limit $M_1^2(\Box) \to 0$. On the contrary, the inclusion of the constraint (3.5) in the action (2.11) with a Lagrangian multiplier — additional scalar field — improves the situation. It leads to the equations of motion which yield as a solution the gravitational potential (3.6) for the following choice of nonlocal operators

$$M_1^2(\Box) = M_2^2(\Box) = \frac{1}{G_4} \frac{\Box - m^2}{\Box}. \tag{3.8}$$

4. DGP model and the strong coupling problem

Effective long-distance modification of the Einstein theory is also provided by the DGP model of brane induced gravity. Its fundamental action includes the five-dimensional (bulk) Einstein term of the metric $G_{AB}(X)$, $A = 0, 1, 2, 3, 5$, and the Einstein term of

the metric $g_{\mu\nu}(x)$, $\mu = 0,1,2,3$, on the 4-dimensional brane viewed either as a boundary of the 5D bulk or as a fixed point of Z_2-identification on the 5D orbifold

$$S_{DGP}[\,G_{AB}(X)\,] = \frac{1}{16\pi G_5}\int d^5X\, G^{1/2}\, R^{(5)}(\,G_{AB}\,) \\ + \int d^4x\, g^{1/2}\left(\frac{1}{8\pi G_5}[K] + \frac{1}{16\pi G_4}\,R(\,g_{\mu\nu}\,)\right). \tag{4.1}$$

The brane term is accompanied by the Gibbons-Hawking surface integral of the trace of extrinsic curvature of the brane $[K]$ (analogous to the one at infinitely remote boundary written in (2.8) in the noncovariant form). The 4-dimensional gravitational constant here $G_4 = 1/M_4^2$ is essentially different from the 5-dimensional one $G_5 = 1/M_5^3$. They both determine two different scales of the DGP model — M_4 typically of the Planckian value and the crossover scale to the infrared regime

$$m = \frac{2M_5^3}{M_4^2} \tag{4.2}$$

(usually identified with the Hubble scale of the Universe $m = H = 10^{-28}$ cm^{-1}).

As is known, the DGP model is analogous to the FP theory with the mass term more soft in the infrared, $m^2 \Rightarrow m\sqrt{-\Box}$, in which the role of m is played by the scale (4.2). To show this, one can build the effective brane action which follows from (4.1) by integrating out the bulk metric $G_{AB}(X)$ subject to fixed boundary data on the brane. This can be done perturbatively. For this purpose expand the fundamental action in gravitational perturbations

$$G_{AB}(X) = \eta_{AB} + H_{AB}(X), \tag{4.3}$$

solve the linear equations for $H_{AB}(X)$ in the bulk and substitute the result back into the quadratic part of the action. We shall work in the coordinate system in which the brane is located at the fixed value of the fifth coordinate, $X^5 \equiv y = 0$. For gauge fixing in the bulk add to the action (4.1) the gauge breaking term quadratic in the linearized de Donder-Fock gauge

$$S_{\rm gauge}[\,H_{AB}] = -\frac{M_5^3}{16\pi}\,\frac{1}{2}\int d^5X\,\eta^{AB}F_AF_B, \tag{4.4}$$

$$F_A = \partial^B H_{AB} - \frac{1}{2}\,\partial_A H. \tag{4.5}$$

In this gauge the bulk equations of motion are most simple and form the following boundary value problem

$$\Box_5 H_{AB}(X) = 0, \tag{4.6}$$

$$H_{AB}(x,y)\,\big|_{y=0} = h_{AB}(x), \;\; h_{AB}(x) \equiv (h_{\mu\nu}(x), N_\mu(x), h_{55}(x)). \tag{4.7}$$

On a flat background $\Box_5 = \Box + \partial_y^2$, and the solution of this problem nonsingular at the bulk infinity can be written down in the following elegant form [13]

$$H_{AB}(x, y) = e^{-y\Delta}\, h_{AB}(x) \tag{4.8}$$

in terms of the auxiliary operator

$$\Delta = \sqrt{-\Box} \tag{4.9}$$

(the case of Lorentzian spacetime we treat as the analytic continuation from the Euclidean space where the operator $\Box$ is negative definite, so that $H_{AB}(x, y)$ vanishes at $y \to \infty$).

Substituting the obtained solution in the 5D part of the DGP action (4.1) $S_5[\,G_{AB}]$ (with the five-dimensional curvature and the Gibbons-Hawking integral) and taking into account the gauge breaking term (4.4) one gets [13]

$$\begin{aligned} S_5[\,G_{AB}] + S_{\rm gauge}[\,H_{AB}] = \frac{M_4^2}{16\pi}\,\frac{m}{4} \int d^4x\, \Big(&-\tilde{h}^{\mu\nu}\Delta\tilde{h}_{\mu\nu} + \frac{1}{2}\tilde{h}\Delta\tilde{h} \\ &+ \tilde{h}\Delta h_{55} - \frac{1}{2}h_{55}\Delta h_{55}\Big)\,, \end{aligned} \tag{4.10}$$

where $m = 2M_5^3/M_4^2$ is the DGP scale (4.2) and $\tilde{h}_{\mu\nu}$ is the following combination of the metric induced on the brane and shift functions in the fifth dimension $G_{5\mu} = N_\mu$

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{\Delta}\,(\partial_\mu N_\nu + \partial_\nu N_\mu). \tag{4.11}$$

Note that this combination is gauge invariant with respect to the four-dimensional coordinate transformations

$$\delta_\xi h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu,\ \ \delta_\xi N_\mu = -\Delta\xi_\mu,\ \ \delta_\xi h_{55} = 0. \tag{4.12}$$

In their turn, these transformations represent the restriction to the brane of the residual gauge transformations in the bulk $\delta_\Xi H_{AB} = \partial_A \Xi_B + \partial_B \Xi_A$,

$$\Xi^\mu(x, y) = e^{-y\Delta}\, \xi^\mu(x),\ \ \ \Xi^5(x, y) = 0, \tag{4.13}$$

which, because of $\Box_5 \Xi^A(x, y) = 0$, leave invariant the de Donder gauge (4.5) and do not shift the brane from its location at $y = 0$, .

Thus, as expected, the effective action (4.10) induced from the bulk is invariant with respect to four-dimensional transformations, but this invariance is actually realized by means of the Stueckelberg fields N_μ, usually introduced by hands for the covariantization of the noninvariant action. By varying (4.10) with respect to N_μ, these fields can be excluded in terms of the metric variables

$$N_\mu = \frac{1}{\Delta}\Big(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h - \frac{1}{2}\partial_\mu h_{55}\Big)\,, \tag{4.14}$$

which leads to the manifestly invariant expression for $\tilde{h}_{\mu\nu}$ in terms of the linearized Ricci tensor

$$\tilde{h}_{\mu\nu} = -2\frac{1}{\Box}R_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{\Box}h_{55}. \tag{4.15}$$

Their substitution to the bulk action leads to

$$S_5 + S_{\rm gauge} = \frac{M_4^2}{16\pi}\, m \int d^4x\, \left(-R^{\mu\nu}\frac{\Delta}{\Box^2}R_{\mu\nu} + \frac{1}{2}R\frac{\Delta}{\Box^2}R - R\frac{\Delta}{\Box}h_{55}\right), \tag{4.16}$$

where the scalar variable h_{55} can no longer be excluded in terms of the metric field — the situation discussed above.

Adding (4.16) to the four-dimensional part of the DGP action, rewritten in the nonlocal form (2.7), we finally obtain the quadratic part of the effective action on the brane

$$S^{\rm eff}_{DGP}[\,g_{\mu\nu},\Pi\,] = -\frac{M_4^2}{16\pi}\int dx\, g^{1/2}\left\{\Big(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\Big)\frac{\Box - m\Delta}{\Box^2}\,R_{\mu\nu} + m\Pi R\right\}, \tag{4.17}$$

where the Lagrangian multiplier to the scalar curvature Π is related to h_{55} component of the 5D metric according to

$$h_{55} = -2\Delta\Pi. \tag{4.18}$$

The variable Π was introduced in [13] as a longitudinal part of the 5D shift function $N_\mu = \partial_\mu\Pi + N'_\mu$. It parameterizes the brane bending in the form of the 5D diffeomorphism of the bulk metric

$$\delta_\Xi H_{5\mu} = \partial_\mu \Xi_5, \tag{4.19}$$

$$\delta_\Xi H_{55} = 2\,\partial_y \Xi_5 = -2\Delta\Pi, \tag{4.20}$$

with the vector field $\Xi^A(x,y) = \delta^A_5\, e^{-y\Delta}\Pi(x)$. Similarly to (4.13) this diffeomorphism does not break the de Donder gauge in the bulk, but it shifts the brane in the direction of the fifth coordinate by $\Xi^5(x,0) = \Pi(x)$ and, therefore, is not the symmetry of the action. It does not manifest itself in the bulk, and its effect reduces to the contribution on the brane, which begins with the local term $m\Pi R$.

Thus, the 4-dimensional effective action of the DGP model does not take the form (2.11), because it contains auxiliary constraint with the Lagrange multiplier which cannot be expressed in terms of metric variables. Therefore, in the linear approximation the DGP model on the brane is effectively described by the Fierz-Pauli theory with the nonlocal mass term of the form (4.16), generated from the bulk. In essence, the expression (4.16) turns out to be the covariant completion of this term (3.1) with the

infrared soft mass [2] $\sqrt{m\Delta}$. Linearized gravitational potential of matter source in this model is analogous to (2.13)

$$h_{\mu\nu} = -16\pi G_4 \frac{1}{\Box - m\Delta} \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T\right) + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu. \tag{4.21}$$

Interaction of matter on the brane is determined here by the propagator which for small spacetime intervals $|x| \ll L = 1/m$, $|\Box| \gg m^2$, obviously coincides with the 4-dimensional one. On the contrary, beyond the crossover scale (4.2) the coupling becomes five-dimensional. This is usually interpreted as a gravitational leakage to the bulk — 4D graviton is metastable and decays within a lifetime $L = 1/m$ similarly to the Gregory-Rubakov-Sibiryakov model [24]. In contrast to GRS model which suffers from ghost negative-energy states [25, 26, 27], the DGP model (like FP theory) is ghost-free.

Similarly to FP model, the constraint term of (4.17) results in the VDVZ problem — tensor structure of the gravitational potential (4.21) differs from general relativity and corresponds to to Fierz-Pauli theory in all ranges of distances. Note that kinematically the variable Π of brane embedding into the bulk is analogous here to the radion mode in the Randall-Sundrum model [28], which guarantees there the recovery of the general relativistic structure. Here, however, this mode does not cope with this task and the DGP model suffers from the VDVZ problem [10].

Another consequence of the constraint term in the DGP action (4.17) is the problem of low strong-coupling scale. Point is that the variable Π has a nature of the Lagrange multiplier only in the quadratic order, while in higher order terms of the weak field expansion it enters the action nonlinearly and gives rise to composite operators built of the powers of Π, N_μ, $h_{\mu\nu}$ and their derivatives. On the other hand, its kinetic term in the quadratic order originates entirely from mixing with $h_{\mu\nu}$ (in view of its Lagrangian multiplier nature in $M_P^2\, m\, \Pi R \sim M_P^2\, m\, \partial\Pi\, \partial h$) and is small because of smallness of m. As a result, after the diagonalization of the full quadratic term the Π-mode acquires the kinetic term $\sim M_P^2 m^2 (\partial\Pi)^2$, and the transition to the canonically normalized field $\hat\Pi$, $\Pi = \hat\Pi/(mM_P)$, when expanding in Π gives rise to higher and higher negative powers of small quantity m [12]. Then, the composite operators of high dimensionalities become suppressed by the factors of the form $1/M_P^p\, m^q$ and get strong at the scale $\Lambda_{p,q} = (M_P^p\, m^q)^{1/(p+q)}$. As shown in [13, 29], the lowest scale occurs for the cubic in Π interaction

$$-\frac{1}{\Lambda^3} \int d^4x\, (\partial\Pi)^2 \Box\Pi, \tag{4.22}$$

[2] Covariant structures of such a type as a realization of the nonlocal cosmological "constant" were also discussed in context of the renormalization theory in [22, 23].

and its value

$$\Lambda = (m^2 M_P)^{1/3} \sim (1000\,\mathrm{km})^{-1} \tag{4.23}$$

is much lower than the submillimeter scale $(0.02\mathrm{mm})^{-1}$ up to which the Newton law is verified by high precision table-top experiments [5]. Analogous situation is well known for the nonlinear FP model (with the full Einstein term and quadratic mass term) — its strong-coupling scale being equal to $\Lambda_5 = (m^4 M_P)^{1/5}$ [12][3].

The recovery of Einstein phase in the DGP theory (with the correct tensor structure of the propagator) can be attained by its synthesis with the Randall-Sundrum model. If the brane has a positive tension fine tuned to the negative cosmological constant in the bulk, then in the perturbative domain below the strong coupling scale, $\Box \ll \Lambda^2$, the gravitational potential (2.13) has operator functions [31]

$$M_1^2(\Box) = M_4^2 \left[1 + \frac{m\,K_1(l\Delta)}{\Delta\,K_2(l\Delta)}\right], \tag{4.24}$$

$$\alpha(\Box) = \frac{2}{2+m\,l} + \frac{2}{3}\,\frac{lm}{2+l\,m}\,\left[1 + \frac{K_1(l\Delta)}{l\Delta\,K_2(l\Delta)}\right], \tag{4.25}$$

in terms of McDonald functions of the first and second order, $K_{1,2}(x)$, and curvature radius l of the AdS bulk. In the distance range $1/m \gg 1/\Delta \gg l$ this potential describes the 4-dimensional general relativistic law with $\alpha(\Box) \simeq 1$ and effective Planck mass $M_P^2 = M_4^2\,(1 + lm/2) \simeq M_4^2$ [31]. Thus, within the hierarchy of the horizon (DGP) and the AdS scales $1/m = 1/H \gg l$ this model does not suffer from the VDVZ problem. Actually, this is a generalization of the well-known result that this problem is absent for spin-2 massive field in (A)dS spacetime [32, 33] with the cosmological constant Λ in the limit $m^2/\Lambda \to 0$.

Another generalization that helps to resolve the VDVZ problem consists in increasing the number of extra dimensions in the DGP model, which has a good motivation from string theory [34]. Correct tensor structure in this case is, however, achieved by the price of ghost states of tachyon nature [26] which indicates classical and quantum instabilities. Two approaches were suggested to avoid these states, based on the necessity to make the UV regularization for branes with codimension $N > 1$ [35]. The regularization smearing the brane was used in [36] (together with the $D = 4 + N$-dimensional scalar curvature replacing on the brane the 4-dimensional one) and the model was shown to be ghost and tachyon free, and, moreover, its 5D version acquiring a big strong-coupling scale $\Lambda_9 = (M_5^7 m^2)^{1/9} \gg \Lambda_3$ much exceeding (4.23) [37].

[3] At the classical level the strong-coupling and VDVZ problems in the FP model may apparently be circumvented by infinite resummation of nonlinear terms [16]. However, at the quantum level the theory has the strong coupling scale [30] which at best can be raised to $(m^2 M_P)^{1/3}$ by the inclusion of higher-dimensional operators [12].

Another approach to DGP models with $N \geq 2$ [38] was based on the interpretation of Green's functions poles different from [26]. The unitarity of the theory was recovered by noting that the ghost tachyon of [26] belongs to the unphysical sheet of complex Mandelstam variable and, apparently, can be removed by the sort of the Lee-Wick prescription in local field theory with ghost states [39]). The accompanying loss of analyticity of the propagator results in the loss of causality at the horizon scale $L = M_4/M_D^2$, which as advocated in [38] does not contradict causality at small and intermediate distance scales.

5. Constrained DGP model

VDVZ and strong coupling problems in the DGP model invalidate the weak field expansion everywhere except the ultrainfrared domain of energies E below the horizon scale $E \ll m = 1/H_0$. This makes the flat-space perturbation theory (and its covariantization according to the lines of Sect.2) inefficient. Moreover, it turns out to be incomplete without the the UV completion at the quantum level, because the infinite tower of higher dimensional operators limits its predictive power even for larger distances. Recently there were several attempts to save the situation [29, 18]. One of them [29] is based on the transition to the dynamically more safe non-flat background on which the nonvanishing brane extrinsic curvature $K_{\mu\nu}$ introduces big kinetic term for the brane bending mode and, therefore, acts like a dilaton field controlling the strength of its interaction at each spacetime point. More radical is the suggestion of [18] to interpret the strong coupling problem as a gauge artifact that can be circumvented by a special modification of perturbation theory called in [18] the constrained perturbation theory.

This modification consists in adding to the action (4.1) the full gauge-fixing term

$$S_{\rm gauge}[H_{AB}] = -\frac{M_4^2}{16\pi}\int d^4x\,\frac{F_\mu^2}{2\sigma} + \frac{M_5^3}{16\pi}\int d^5X\left(\frac{1}{2\gamma}B_5^2 - \frac{1}{2\beta}B_\mu^2\right), \tag{5.1}$$

which includes the set of special 5-dimensional gauges in the bulk

$$B_\mu = \partial_\mu H_{55} - \partial^\alpha H_{\alpha\mu}, \;\; B_5 = \partial^\mu H_{\mu 5}, \tag{5.2}$$

along with the 4-dimensional harmonic gauge for the intrinsic metric $h_{\mu\nu}$ on the brane

$$F_\mu = \partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h. \tag{5.3}$$

From the viewpoint of gauge-fixing procedure the brane gauge term is redundant, because the bulk term in (5.1) alone completely fixes all coordinate invariances of

the theory[4]. Therefore this term was interpreted in [18] as an additional constraint extending beyond conventional gauge-fixing procedure. The calculations show that for particular limiting values of "gauge-fixing" parameters $\beta \to 0$, $\gamma \to 0$ and $\sigma = 1$ the gravitational potential of matter sources on the brane does not suffer from strong coupling and VDVZ problems. In particular, in Eq.(3.6) it has the operator coefficient $\alpha(\Box)$ interpolating between the FP value $\alpha_{FP} = 2/3$ in the deep IR regime $\Delta \equiv \sqrt{-\Box} \ll m$ and the Einstein value $\alpha = 1$ for short distances $\Delta \gg m$ and does not have terms singular for $m \to 0$.

This result, however, can hardly be interpreted as a proper solution of these problems in the DGP model. Rather than being the constrained perturbation theory for this model, the suggestion of [18] represents the modification of the model itself, and this modification is not unique. Indeed, the theory with the action (4.1) modified by the term (5.1) in the limit

$$\beta \to 0, \quad \gamma \to 0, \tag{5.4}$$

but with an arbitrary parameter σ, has the following gravitational potential on the brane, $h_{AB} = H_{AB}|$, (in what follows the vertical bar denotes the restriction of a bulk quantity to the brane)

$$h_{\mu\nu} = -\frac{16\pi}{M_4^2}\,\frac{1}{\Box - m\Delta}\left(T_{\mu\nu} - \frac{1}{2}\,\alpha_\sigma(\Box)\eta_{\mu\nu}T\right) + \frac{16\pi}{M_4^2}\,\partial_\mu\partial_\nu\frac{\omega_\sigma(\Box)}{\Box}\,T, \tag{5.5}$$

$$h_{55} = \frac{16\pi}{M_4^2}\,\frac{1}{2}\,\frac{\Box - 2\sigma m\Delta}{\Box - 4m\Delta - 3\sigma m^2}\,\frac{1}{\Box}\,T, \tag{5.6}$$

$$h_{5\mu} = 0 \tag{5.7}$$

with the operator coefficients (see the derivation in Appendix)

$$\alpha_\sigma(\Box) = \frac{\Box - 3m\Delta - 2\sigma m^2}{\Box - 4m\Delta - 3\sigma m^2}, \tag{5.8}$$

$$\omega_\sigma(\Box) = \frac{1}{\Box - m\Delta}\,\frac{(1-\sigma)\,m\Delta}{\Box - 4m\Delta - 3\sigma m^2}. \tag{5.9}$$

The data (5.5)-(5.7) is propagated from the brane into the bulk by the massless Klein-Gordon equation[5] (4.6) according to (4.8) and does not have dangerous parts blowing at $m \to 0$.

[4]The residual gauge transformations for the bulk gauge (5.2), $\delta H_{AB} = 2\partial_{(A}\Xi_{B)}$, are locally parametrized by *one* scalar field Ξ_5 propagating in the bulk, $\Box_5\Xi_5 = 0$, $\Xi_\mu = \partial_\mu(1/\Box)\partial_5\Xi_5$, but the corresponding diffeomorphism should not move the brane, $\Xi_5| = 0$. Therefore, with this condition on the brane and zero boundary conditions at infinity $\Xi_5(X)$ vanishes identicallty everywhere in the bulk and thus does not leave any gauge freedom.

[5]This is a corollary of the limit (5.4). For nonvanishing β and γ only the transverse-traceless, $H^{TT}_{\mu\nu}$, and conformal, $\eta_{\mu\nu}\Phi$, sectors of H_{AB} are subject to this propagation law (see Eqs.(A.13)-(A.15) in Appendix).

The expression (5.5) for on-brane metric can be obtained from the effective action of the form (2.11) with the nonlocal form factors

$$M_1^2(\Box) = \frac{1}{G_4}\,\frac{\Box - m\Delta}{\Box}, \quad M_2^2(\Box) = \frac{1}{G_4}\,\frac{\Box - 2m\Delta - \sigma m^2}{\Box}. \tag{5.10}$$

This action, in contrast to (4.17), does not contain any constraint term, because no strong coupling mode is present and all variables express in terms of the 4-dimensional metric. Since the constrained theory is not gauge-invariant, the longitudinal term of the solution (5.5) is not ambiguous just like in the Fierz-Pauli model with (3.7). Therefore, this solution can be regarded as obtained from the *covariant* effective action (2.11) in a particular σ-dependent gauge

$$\partial^\nu h_{\mu\nu} - \frac{1}{2}\frac{\Box - 2\sigma m\Delta}{\Box - \sigma m\Delta}\partial_\mu h = 0. \tag{5.11}$$

This gauge interpolates between the Fierz-Pauli gauge (3.4) in the infrared and the harmonic gauge for high energies, just like the operator coefficient (5.8) interpolates between the corresponding Fierz-Pauli and Einstein values.

Thus, for generic nonsingular σ the brane metric (5.5) has the Einstein high-energy limit and the (soft-mass) Fierz-Pauli behavior at low energies. But at intermediate energies it corresponds to the one-parameter family of physically inequivalent theories. In particular, they are not equivalent to the original DGP model, because their equations are spoiled by gauge terms which are nonvanishing even on shell. For example, one of the bulk Einstein equations in the DGP model implies the vanishing 4-dimensional scalar curvature, $\mathcal{E}_{55}^{(5)} \sim R^{(4)} = 0$, while for (5.5) it reads

$$R^{(4)} = -\frac{8\pi}{M_4^2}\,\frac{1}{\Box - 4m\Delta - 3\sigma m^2}\,T \tag{5.12}$$

and goes to zero only in the singular limit $\sigma \to \infty$ (this limit obviously corresponds to the original DGP model and is marred by the strong-coupling and VDVZ problems).

The constrained theories depend not only on the σ, but also on the bulk "gauge-fixing" parameters β and γ. This gauge dependence is an artifact of introducing the redundant brane "gauge" in (5.1). As shown in Appendix, the Israel junction condition which underlies the derivation of (5.5)-(5.7) establishes the relation (A.23) between this gauge F_μ and the boundary value of the bulk gauge $B_5\,|$. Only their combination (A.23) is vanishing on shell, so that separately $F_\mu(x)$ and $B_A(X)$ are nonzero[6]. Therefore these constraints go beyond a consistent gauge-fixing procedure and modify the original model.

[6]Brane gauge F_μ can be separately put to zero by demanding the continuity of $H_{5\mu}$-coefficients across the brane[40], but this requirement seems contrived, because it does not follow from the geometrically invariant formulation of the Z_2-symmetry of brane orbifolds.

6. Discussion and conclusions

The above discussion shows that the strong-coupling and VDVZ problems are very robust within covariant infrared modifications of Einstein theory. In particular, for DGP model the VDVZ problem is a direct corollary of the 5-dimensional Einstein equations. The (55)-component of these equations implies the vanishing of the 4-dimensional scalar curvature (3.5) which means that there is no conformal term $\eta_{\mu\nu}\varphi$ in the transverse-traceless decomposition of the metric potential on the brane $h_{\mu\nu} = h^{TT}_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$. Therefore, up to gauge transformation terms the linearized gravitational field is given by the transverse-traceless part of the matter stress tensor, $h_{\mu\nu} \sim T^{TT}_{\mu\nu}$, and this unambiguously fixes the α-coefficient of (2.13) at the Fierz-Pauli value 2/3, because

$$T^{TT}_{\mu\nu} = T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T + \frac{1}{3}\partial_\mu\partial_\nu\frac{1}{\Box}T. \tag{6.1}$$

Thus, the VDVZ problem in DGP model cannot be circumvented without spoiling the basic dynamical equations in the bulk. This is exactly what happens in the constrained DGP model of [18].

The strong coupling problem also has a gauge-invariant origin, even though the derivation of Sect.4 seems to point at the particular harmonic gauge as a source of this difficulty. Note that in the derivation of (4.17) we could not exclude via the variational equation the h_{55} metric component which turned out to be the strongly coupled mode (4.18) — the Lagrangian multiplyer of the scalar curvature constraint in (4.17). The impossibility to solve the linearized Einstein equations in the bulk for lapse and shift functions $N_A = G_{5A}$ follows from the degeneracy of the operator

$$S^{AB}\delta(X,Y) = \frac{\delta^2 S_5[\,G\,]}{\delta N_A(X)\,\delta N_B(Y)}. \tag{6.2}$$

On flat space background it is really degenerate because its $(5A)$-column is identically zero and the $(\mu\nu)$-block, $S^{\mu\nu} = \partial^\mu\partial^\nu - \eta^{\mu\nu}\Box$, has as a zero mode an arbitrary longitudinal vector $N_\nu = \partial_\nu\Pi$ — the (5ν)-component of the strong coupling mode (4.19).

At this point it is useful to compare the gauge (in)dependence properties of the original DGP model and its constrained version of Sect.5. The de Donder gauge F_A, (4.5), used in Sect.4 is qualitatively different from the gauge B_A in (5.2). It belongs to the class of relativistic gauges for which the residual transformations are locally parametrized by the full set of five propagating modes, $\Box_5\Xi_A = 0$, and the Faddeev-Popov operator $\Box_5$ is dynamical for all components of Ξ_A. The fifth component Ξ_5 is ruled out by Dirichlet boundary conditions on the brane $\Xi_5| = 0$ (because the movement of the brane is not a gauge transformation), and the residual transformations reduce to 4-dimensional diffeomorphisms (4.13). Their fixation requires four extra gauges

imposed on the brane, and this is a main point of departure from the special gauge (5.2) of [18]. In contrast to [18] these extra gauges do not violate Ward identities and do not change the physical contents of the model. This property is based on the fact that, though the effective brane action (4.10) is gauge dependent, its gauge dependence gets eradicated by the transition to the effective action (4.17) with on-shell values of the functions N_A. This is different from the constrained version of the DGP model for which the effective form factors (5.10) explicitly depend on all – bulk and brane – "gauge-fixing" terms.

To clarify the properties of the action (4.10) versus (4.17), note that the former was obtained as a fundamental DGP action

$$S^{\rm eff}_{DGP}[\,g_{\mu\nu},\,N_A\,] = S_{DGP}[\,G_{AB}\,]\,\Big|_{G_{AB}=G_{AB}[\,g_{\mu\nu},\,N_A\,]} \tag{6.3}$$

$$g_{\mu\nu} = G_{\mu\nu}\big|\,,\;\; N_A = G_{5A}\big|\,, \tag{6.4}$$

evaluated on the solution of the bulk Einstein equations $G_{AB}[\,g_{\mu\nu}, N_A\,]$ subject to independent boundary conditions for *all* five-dimensional metric coefficients on the brane. The action functional for such boundary conditions depends on the choice of gauge, because its on-shell restriction is not complete regarding the boundary values $N_A(x)$ of $G_{5A}(X)$-coefficients. Only their exclusion by virtue of variational equations

$$\frac{\delta S^{\rm eff}_{DGP}}{\delta N_A(x)} = 0 \tag{6.5}$$

in terms of the metric $g_{\mu\nu}$, $N_A = N_A[\,g_{\mu\nu}]$, makes the action gauge independent[7]. However, the strong-coupling problem prevents from solving these equations for all $N_A = N_A[\,g_{\mu\nu}]$ – only its transverse μ-components can be excluded, and at least one scalar mode Π remains off shell among the arguments of (4.17). Nevertheless, the result turns out to be covariant and bulk-gauge independent, though suffering from the strong coupling and VDVZ problems.

In essence, the gauge difficulties with these problems follow from the lack of manifestly covariant and gauge independent formalism. The on-shell reduction which makes the brane action gauge independent does not resolve this problem. In particular, the construction of brane-to-brane propagator requires the off-shell extension of the brane action and, therefore, is sensitive to the choice of gauge. Moreover, the off-shell extension of the action is necessary at the quantum level, where it should be integrated

[7] In the relativistic gauge $F_A(X)$, (4.5), the variational equations (6.5) represent the requirement of zero boundary data for these gauge conditions, $F_A| = 0$. In the bulk they satisfy the homogeneous equation $\Box_5 F_A(X) = 0$ (with the Faddeev-Popov operator $\Box_5$). Therefore $F_A(X) = 0$ everywhere in the bulk and the on-shell contribution of the gauge-breaking term (4.4) vanishes and makes the whole on-shell action gauge independent.

over the fields on the brane to generate the full set of Feynman diagrams. This integral in which the bulk integration is done first, while the integration over brane fields is reserved for the last, schematically looks like

$$\int D_5 G_{AB}\, e^{-S_{DGP}[G_{AB}]}\,(...)$$
$$= \int D_4 g_{\mu\nu}\, D_4 N_A\, e^{-S^{\rm eff}_{DGP}[g_{\mu\nu},\,N_A]-\mathit{\Gamma}_{\rm loop}[g_{\mu\nu},\,N_A]}\,(...). \qquad (6.6)$$

Here ellipses denote all the details of gauge-fixing procedure (including the contribution of relevant Faddeev-Popov determinants), the subscript of D in the integration measure indicates the spacetime dimensionality of fields — path integral over the 5-dimensional metric $G_{AB}(X)$ in the bulk versus 4-dimensional fields $g_{\mu\nu}(x)$ and $N_A(x)$ on the brane. $\mathit{\Gamma}_{\rm loop}[\,g_{\mu\nu},\,N_A\,]$ denotes the loop part of the brane effective action following from the functional integration over bulk fields bounded by brane fields $g_{\mu\nu}(x)$ and $N_A(x)$.

In this form the functional integral is not manifestly covariant, because the integrand on the right-hand side is off shell and, therefore, gauge dependent. Its overall gauge independence is not manifest because it is attained only as a result of brane field integration. However, the strategy of its calculation can be improved by including the integration over $D_4 N_A$ into the bulk, so that (6.6) takes the form

$$\int D_4 g_{\mu\nu}\, e^{-S^{\rm eff}[g_{\mu\nu}]-\mathit{\Gamma}_{\rm loop}[g_{\mu\nu}]}\,(...) \qquad (6.7)$$

with the brane tree-level $S^{\rm eff}[\,g_{\mu\nu}\,]$ and loop effective $\mathit{\Gamma}_{\rm loop}[g_{\mu\nu}]$ actions *gauge independent off shell*. Such improvement, the corresponding Feynman diagrammatic technique and its gravitational Ward identities will be presented in the coming paper [41], and this technique is expected to establish in a simple way the covariant gauge-independent status of the strong-coupling problem[8].

To summarize, we extended the suggestion of [9] for infrared modifications of Einstein theory to the general case characterized by two nonlocal operators in Ricci tensor and scalar quadratic forms and considered restrictions on these operators imposed by the absence of VDVZ discontinuity. In models with low strong-coupling scale we showed that their effective action also contains the strongly coupled mode which cannot be perturbatively integrated out in terms of the metric variables. It serves in the lowest order as a Lagrange multiplyer for the constraint on metric variables which enforces the curvature scalar to vanish on shell. We demonstrated this situation on the examples of FP, DGP models and the recently suggested constrained version of the DGP model.

[8]Depending on gauge-fixing procedure for both bulk and brane diffeomorphisms the diagrammatic technique for this integral can be essentially simplified by factorizing the bulk and brane-to-brane parts [41], which allows one efficiently to disentangle the different brane and bulk scales of the model.

A. Gravitational potentials in the constrained DGP model

Bulk equations of motion for the action (4.1) modified by the term (5.1) read

$$\mathcal{E}^{(5)}_{\mu\nu} + \frac{1}{\beta}\partial_{(\mu}B_{\nu)} = 0, \tag{A.1}$$

$$\mathcal{E}^{(5)}_{5\mu} + \frac{1}{2\gamma}\partial_{\mu}B_5 = 0, \tag{A.2}$$

$$\mathcal{E}^{(5)}_{55} - \frac{1}{\beta}\partial_{\mu}B^{\mu} = 0, \tag{A.3}$$

where $\mathcal{E}_{AB} = R_{AB} - \frac{1}{2}G_{AB}R^{(5)}$ is the 5-dimensional Einstein tensor. Bianchi identities $\nabla^B\mathcal{E}^{(5)}_{AB} = 0$ then yield the equations of motion for gauge conditions

$$\Box_5 B_5 = 0, \tag{A.4}$$

$$B_\mu = -\frac{\beta}{2\gamma}\partial_\mu\frac{1}{\Box}\partial_5 B_5, \tag{A.5}$$

in which only B_5 is propagating in the bulk, while B_μ express algebraically in terms of B_5.

From (A.2)-(A.3) one gets the $5A$-components of the metric

$$H_{5\mu} = \frac{\partial_5}{\Box}(\partial^\nu H_{\mu\nu} - \partial_\mu H) - (\gamma+1)\,\partial_\mu\frac{\partial_5}{\Box^2}(\partial\partial H - \Box H), \tag{A.6}$$

$$H_{55} = \frac{1}{\Box}\partial\partial H - \frac{1}{2}\beta\frac{1}{\Box}(\partial\partial H - \Box H), \tag{A.7}$$

whence

$$B_5 = \partial^\mu H_{5\mu} = -\gamma\,\frac{\partial_5}{\Box}(\partial\partial H - \Box H), \quad B_\mu = \frac{1}{2}\beta\partial_\mu\frac{\partial_5^2}{\Box^2}(\partial\partial H - \Box H), \tag{A.8}$$

where $\partial\partial H \equiv \partial^\mu\partial^\nu H_{\mu\nu}$ and $H \equiv \eta^{\mu\nu}H_{\mu\nu}$. Combining (A.5), (A.8) and the definition of B_μ one has

$$\partial^\mu B_\mu = \Box h_{55} - \partial\partial H = \frac{\beta}{2}\frac{\partial_5^2}{\Box}(\partial\partial H - \Box H). \tag{A.9}$$

Together with (A.7) this implies

$$\Box_5(\partial\partial H - \Box H) = 0. \tag{A.10}$$

The trace of (A.1) then gives

$$\Box_5 H - (2\gamma + \beta/2)(\partial\partial H - \Box H) = 0. \tag{A.11}$$

With the 4-dimensional transverse-traceless decomposition of $H_{\mu\nu}$ ($\eta^{\mu\nu} H^{TT}_{\mu\nu} = 0$, $\partial^\nu H^{TT}_{\mu\nu} = 0$, $\partial^\mu \varphi_\mu = 0$)

$$H_{\mu\nu} = H^{TT}_{\mu\nu} + \partial_\mu \varphi_\nu + \partial_\nu \varphi_\mu + \eta_{\mu\nu}\Phi + \partial_\mu \partial_\nu \Psi, \tag{A.12}$$

the gauge becomes $B_\mu = -\Box \varphi_\mu + \partial_\mu (h_{55} - \Phi - \Box\Psi)$. On the other hand, from (A.5) and (A.8) it equals $B_\mu = 3\beta \partial_\mu \varphi/2$. Comparison of these expressions gives $\varphi_\mu = 0$.

Thus, finally from (A.10), (A.11) and the transverse-traceless part of (A.1) we have the set of bulk equations of motion for nonvanishing metric components

$$\Box_5 H^{TT}_{\mu\nu} = 0, \tag{A.13}$$

$$\Box_5 \Phi = 0, \tag{A.14}$$

$$\Box_5 \Psi + 3(2\gamma + \beta/2)\Phi = 0 \tag{A.15}$$

and

$$H_{5\mu} = 3\gamma \frac{\partial_5}{\Box} \partial_\mu \Phi, \quad H_{55} = (1 + 3\beta/2)\Phi + \Box\Psi. \tag{A.16}$$

Linearized Israel junction condition for the action (4.1) with the additional term (5.1) (in the presence of conserved matter sources on the brane) reads

$$m(K_{\mu\nu} - \eta_{\mu\nu} K) - \mathcal{E}^{(4)}_{\mu\nu} + \frac{1}{2\sigma}(\partial_\mu F_\nu + \partial_\nu F_\mu - \eta_{\mu\nu} \partial_\alpha F^\alpha) + \frac{8\pi}{M_4^2} T_{\mu\nu} = 0, \tag{A.17}$$

where $K_{\mu\nu} = (1/2)(\partial_5 H_{\mu\nu} - \partial_\mu H_{\nu 5} - \partial_\nu H_{\mu 5})$ is the extrinsic curvature of the brane. Transverse-traceless part of (A.17) gives

$$(m\partial_5 + \Box) H^{TT}_{\mu\nu}\Big| = -\frac{16\pi}{M_4^2} T^{TT}_{\mu\nu}, \tag{A.18}$$

while the longitudinal and trace parts imply

$$\left(3m\partial_5 + \frac{1}{\sigma}\Box\right)\Phi\,\Big| - \frac{1}{2\sigma}\Box^2 \Psi\,\Big| = 0, \tag{A.19}$$

$$\left((9\gamma - 6)m\partial_5 + \Big(\frac{1}{2\sigma} - 3\Big)\Box\right)\Phi\,\Big| - \frac{1}{2}\left(3m\partial_5 + \frac{1}{\sigma}\Box\right)\Box\Psi\,\Big| + \frac{8\pi}{M_4^2} T = 0. \tag{A.20}$$

For $\beta = \gamma = 0$ the model essentially simplifies. Equations (A.14) and (A.15) decouple and under zero boundary conditions at $y = \infty$ yield

$$\Phi(x, y) = e^{-y\Delta}\Phi\,\Big|, \quad \Psi(x, y) = e^{-y\Delta}\Psi\,\Big|, \tag{A.21}$$

so that everywhere in equations (A.18)-(A.20) ∂_5 can be replaced by the (nonlocal) operator on the brane $-\Delta$. Their solution on the brane becomes

$$\Phi\,\Big| = \frac{8\pi}{3M_4^2} \frac{1}{\Box - 4m\Delta - 3\sigma m^2} T, \quad \Psi\,\Big| = 2\frac{\Box - 3\sigma m\Delta}{\Box^2}\Phi\,\Big|, \tag{A.22}$$

which finally leads to the gravitational potential (5.5)-(5.9).

Note that the (redundant from the gauge-fixing viewpoint) brane gauge term in (A.17) precludes from enforcing the bulk gauges and, thus, gives rise to gauge dependence discussed in Sect.5. Taking the 4-dimensional divergence of the Israel junction condition (A.17) and using the Gauss-Codazzi equation $\nabla^\mu(K_{\mu\nu} - \eta_{\mu\nu}K) = R^{(5)}_{5\nu}$ one has in virtue of (A.2)

$$\frac{1}{\gamma}\partial_\mu B_5 \Big| = \frac{1}{m\sigma}\Box F_\mu. \tag{A.23}$$

Thus, the equation for gauge conditions in the bulk (A.4) has nonzero data on the brane and has a nonvanishing solution. Therefore bulk gauge conditions are not enforced, and their contribution to equations of motion (A.1)-(A.3) (proportional to B_5/γ) is nonvanishing even on shell. This situation holds also in the limit of $\gamma \to 0$.

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